Zero and One-dimensional Probes
with N=8 Supersymmetry

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We study the symmetry structure of $N = 8$ quantum mechanics, and apply it to the physics of D0-brane probes in type I' string theory. We focus on the theory with a global $Spin(8)$ R symmetry which arises upon dimensional reduction from $2d$ field theory with $(0, 8)$ supersymmetry. There are several puzzles involving supersymmetry which we resolve. In particular, by taking into account the gauge constraint and central charge we explain how the system preserves supersymmetry despite having different numbers of bosonic and fermionic variables. The resulting zero-point energy leads to a linear potential consistent with supersymmetry, and the metric is largely unconstrained. We discuss implications for type I' string theory and the matrix model proposal for M theory.

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1. Introduction

One of the most interesting phenomena uncovered by string duality is the relation between gauge dynamics and compactification geometry. A particularly simple manifestation of this correspondence occurs in the context of brane probes. It first appeared in [1,2] and was developed in [3-11].

In perturbative string theory, there is a detailed correspondence between worldsheet physics and spacetime physics. In particular, worldsheet supersymmetry is related to spacetime supersymmetry (though the two are not identical). Modular invariance is equivalent to spacetime anomaly cancellation. For large compactification volume the target space of the worldsheet sigma model is the compactification manifold.

It is of some interest to understand the analogues of these statements for probes of dimension $p \neq 1$. Many new quantum field theory results have been obtained on brane probes using the relation to spacetime physics. In each case, the probe theory reflects the geometry of spacetime in its Lagrangian (through for example the metric appearing in the kinetic term for scalar fields). One can consider probes of different dimension in the same background, all of which encode the same spacetime geometry. In this paper we will clarify this correspondence for $p = 1$ and $p = 0$ (with 8 supersymmetries in both cases). For $p = 1$, with $(0, 8)$ supersymmetry, we find that the gauge fermions which are central to the spacetime anomaly cancellation mechanism of the heterotic string are necessary in the D string formalism to cancel worldsheet gauge anomalies which arise when more than one string are present.

We then study the $p = 0$ system that can be thought of as the dimensional reduction of the $(0, 8)$ $p = 1$ system. Aspects of this system were studied in [12-14] and in the matrix model context in [15-18]. At first sight, this system has several puzzling features. In particular, as noted by [12], the number of bosonic and fermionic quantum-mechanical variables is not equal, despite the unbroken supersymmetry. The deficit between the fermionic and bosonic variables leads to a nontrivial potential for one of the fields. In section 3 we explain how this is consistent with the supersymmetry algebra by keeping track of the crucial role played by the gauge constraint. We also find that supersymmetry imposes a very weak constraint on the kinetic terms. Direct computation in the type I' system confirms the presence of a nontrivial metric even when the dilaton tadpole cancels locally.

In the next section we study the two dimensional theory with $(0, 8)$ supersymmetry and in section 3 we study its dimensional reduction to quantum mechanics. In section 4

we consider applications of this quantum mechanical system to the type I' theory and in section 5 to the Matrix model of M theory.

2. The $(0, 8)$ Supersymmetric Theory in $d = 2$

Let us begin by considering 1-brane probes with $(0, 8)$ supersymmetry. One example is the heterotic string (which in type I language is the D-1-brane). The multiplets are as follows: There is a gauge multiplet containing a gauge boson A_μ and 8 left-moving fermions $\lambda_{-,a}$ $(a = 1, \ldots, 8)$. There are two types of matter multiplets. One type includes 8 (nonchiral) bosons X^i, $i = 1, \ldots, 8$ and 8 right-moving fermions $\theta_{+,\dot{a}}$, $\dot{a} = 1, \ldots, 8$. The other type involves only left moving fermions χ_-.

We limit the discussion to matter fields in real representations of the gauge group, because these are the kinds of matter fields which we will find in the applications below. The Lagrangian determined by supersymmetry and gauge invariance is

$$
\mathcal{L} = \frac{-i}{g^2} \lambda_{-,a} \mathcal{D}_+ \lambda_{-,a} + \frac{1}{g^2} Tr F_{\mu\nu}^2 + \partial_\mu X_i \partial^\mu X_i - i\theta_{+,\dot{a}} \mathcal{D}_- \theta_{+,\dot{a}}
$$
$$
+ 2i\theta_{+,\dot{a}} \lambda_{-,a} \sigma_{a\dot{a}}^i X_i - \frac{g^2}{4} \sum_{\alpha,i,j} (X_i T^\alpha X_j)^2 + i\bar{\chi}_r \mathcal{D}_+ \chi_r - \sum_r \bar{\chi}_r m_r \chi_r
$$

(2.1)

Here α runs over the generators of the gauge group, and T^α is the gauge generator in the representation under which the $X_i, \theta_{\dot{a}}$ transform. The supersymmetry transformation laws are:

$$
\delta \lambda_a^\alpha = F_{01}^\alpha \epsilon_a - \frac{g^2}{4} X_i T^\alpha X_j \sigma_{ab}^{ij} \epsilon_b
$$
$$
\delta A_0 = -i\epsilon_a \lambda_a
$$
$$
\delta A_1 = i\epsilon_a \lambda_a
$$
$$
\delta \theta_{\dot{a}} = (\mathcal{D}_+ X_j) \sigma_{\dot{a}a}^j \epsilon_a
$$
$$
\delta X_i = i\epsilon_a \sigma_{a\dot{a}}^i \theta_{\dot{a}}
$$

(2.2)

This theory has an $Spin(8)$ global R symmetry (to be identified with part of the spacetime Lorentz group) under which the supercharges transform in the $\mathbf{8_s}$, $\lambda_{-,a}$ transforms in the $\mathbf{8_s}$, X^i transforms in the $\mathbf{8_v}$, and $\theta_{+,\dot{a}}$ transforms in the $\mathbf{8_c}$. The fact that $\lambda_{-,a}$ and $\theta_{+,\dot{a}}$ transform as spinors of opposite chirality is enforced by the Yukawa coupling between X^i, $\lambda_{-,a}$ and $\theta_{+,\dot{a}}$.

At one loop the field content is constrained by anomalies, as also noted in this context in [18]. Let us denote by r_g, r_R, and r_L the representations of the (λ_-, A_μ), (X, θ_+) and χ_-

multiplets respectively under the gauge symmetry. The cancellation of the gauge anomaly requires

$$8C_2(r_g) + C_2(r_L) = 8C_2(r_R). \tag{2.3}$$

In the case of n heterotic strings, the gauge group is $Spin(n)$. The (X, θ_+) multiplet is in the symmetric tensor and the 32 χ_- multiplets are in the fundamental. Here we are counting Majorana-Weyl fermions. In (2.1) these are grouped into sixteen complex fermions. In the $2d$ context then, anomaly cancellation explains the presence of left-moving fermions which lead to the spacetime $Spin(32)$ gauge symmetry.

3. $N = 8$ Quantum Mechanics

The dimensional reduction of the $(0, 8)$ theory considered above gives an $N = 8$ supersymmetric quantum mechanics with an $Spin(8)$ global R symmetry. This describes the dynamics of type I' zero branes. By contrast, one could also consider theories with (4,4) supersymmetry in $2d$. The dimensional reduction of this again has $N = 8$ supersymmetry but with global R symmetry $Spin(3) \times Spin(5) \subset Spin(8)$. It describes for example the dynamics of D0 branes near D4 branes in type IIA string theory [5].

We will be interested in the $N = 8$ supersymmetric quantum mechanics with $Spin(8)$ global symmetry. Let us begin by considering a theory with only a $U(1)$ gauge multiplet. We will later realize this as the low-energy effective theory obtained by integrating out degrees of freedom from a larger theory. This multiplet contains a gauge boson A_0, a scalar ϕ and eight fermions λ_a, $a = 1, \ldots, 8$. Consider the Lagrangian

$$\mathcal{L} = \int dt \left[f(\phi)\dot{\phi}^2 - if(\phi)\lambda_a \dot{\lambda}_a - k\phi - kA_0 \right] \tag{3.1}$$

The last term here is a $d = 1$ Chern-Simons term. For gauge invariance under large gauge transformations the coefficient k must be an integer. In canonical quantization we pick the gauge $A_0 = 0$ and impose Gauss law $G = \frac{\delta \mathcal{L}}{\delta A_0} = 0$. The Chern-Simons term contributes a linear shift proportional to k to the electric charge G. It corresponds to a background electric charge k. Note that in the system based on (3.1), there are no charged fields and therefore there is no solution to this Gauss law constraint. Also, because of the linear potential the energy is not bounded from below. These issues will be addressed further below.

Naively this theory does not appear supersymmetric. The numbers of bosonic and fermionic fields do not agree. Also, there is a linear potential in ϕ, which is typically forbidden in theories with 8 supercharges (such as $N = 2$ theories in $4d$ and the more closely related $(0, 8)$ models in $2d$). In fact the Lagrangian (3.1) is invariant under the following supersymmetry transformations:

$$\delta A_0 = -i\epsilon_a \lambda_a$$
$$\delta \phi = i\epsilon_a \lambda_a \qquad\qquad (3.2)$$
$$\delta \lambda_a = \epsilon_a \dot{\phi} - i\frac{f'(\phi)}{2f(\phi)}\epsilon_b \lambda_b \lambda_a.$$

It is easy to check that the supersymmetry algebra closes (using the equations of motion). Thus, SUSY puts no constraint on the metric function $f(\phi)$.

The linear potential manages to be supersymmetric due to the Chern-Simons term, which can only appear at one loop order. (In the type I' quantum mechanics we will see that such a term indeed appears out on a flat direction.) The kinetic terms depend in an arbitrary fashion on the scalar ϕ (in particular they are not protected from loop corrections). These features are reminiscent of $N = 1$ supersymmetry in $4d$.

Let us now consider adding q fermionic multiplets charged under the $U(1)$. Each contains a complex fermion χ_r, $r = 1, \ldots, q$. The Lagrangian now contains

$$\mathcal{L}_\chi = -i\bar{\chi}_r \dot{\chi}_r - \bar{\chi}_r \phi \chi_r - \bar{\chi}_r A_0 \chi_r - \sum_{r=1}^{q} m_r \bar{\chi}_r \chi_r. \qquad (3.3)$$

Here we have included bare masses m_r as well as the minimal coupling to the gauge multiplet. The χs do not transform under supersymmetry, and these terms preserve the supersymmetry of the Lagrangian due to the cancellation between the supersymmetry variations of ϕ and A_0 in (3.2)[15]. Notice that integrating out the χs gives contributions to the linear potential in (3.1).

How does this all work in the Hamiltonian formulation? The key fact is that the supercharges anticommute to the Hamiltonian only up to gauge transformations. Explicitly,

$$\{Q_a, Q_b\} = \left(H + \phi G + Z\right)\delta_{ab} \qquad (3.4)$$

where G is the gauge constraint obtained by varying the Lagrangian with respect to A_0, and Z is a central term. The Chern-Simons term in (3.1) induces a shift by $-k\phi$ in the

ϕG term. The linear potential term in (3.1) introduces a term $k\phi$ in H. These cancel out in $\{Q_a, Q_b\}$. A similar calculation explains the terms involving the χs. The gauge contribution ϕG to (3.4) contains a term $-\bar{\chi}_r \phi \chi_r$. This cancels the term $\bar{\chi}_r \phi \chi_r$ in H. Let us now consider the mass terms for the χs, the terms $\bar{\chi}_r m_r \chi_r$. These appear in the Hamiltonian. In order to preserve supersymmetry, they must also appear with a negative sign in the central charge Z. So although the χs do not appear in the supercharges, they appear in the Hamiltonian, gauge constraint, and central charge in such a way as to cancel in $\{Q_a, Q_b\}$.

There is a simple way to integrate out the fermions. Their Lagrangian is of the form $\bar{\chi}(i\partial_t - gV)\chi$ where gV includes the mass term, ϕ and A_0. Integrating out χ leads to $\log \det(i\partial_t - gV)$. Differentiating with respect to g this is $\int dt G(t,t) V(t)$ where $G(t,s) = \exp i \int_s^t gV(u) du \, \theta(t-s)$ is the Green's function of $i\partial_t - gV$. Using $\theta(0) = \frac{1}{2}$, we learn that the Lagrangian resulting from integrating out the fermions is $\frac{1}{2} gV$.

This factor of $\frac{1}{2}$ here is significant. It shows that our quantum mechanical system with a single χ leads to a Chern-Simons term with half integral coefficients. Equivalently, the fermion determinant is not gauge invariant under large gauge transformations. These change the sign of the fermion determinant leading to a global anomaly [1] This problem can be fixed by adding a bare Chern-Simons term with half integral coefficient.

More explicitly, we consider a $U(1)$ gauge theory with q fermions with masses m_r and a bare term $k(\phi + A_0)$. Superficially, we need to impose $k \in \mathbf{Z}$. However, integrating out the fermions we find in the effective action the terms

$$-k(\phi + A_0) + \frac{q}{2}\text{sign}(\phi - m_r)(\phi - m_r + A_0). \tag{3.5}$$

Therefore, for consistency we need

$$k + q/2 \in \mathbf{Z}. \tag{3.6}$$

In other words, the theory with odd q without a bare potential is inconsistent – not gauge invariant. This anomaly can be cancelled by adding a bare term $k(\phi + A_0)$ with $k + 1/2 \in \mathbf{Z}$, which is also not gauge invariant.

It is easy to understand this anomaly from a canonical quantization perspective. Consider $2q$ free real fermions. The global symmetry is $spin(2q)$. The Hilbert space is 2^q

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[1] For a similar global anomaly in quantum mechanics see, e.g. [19].

dimensional in the two spinors of $spin(2q)$. Decomposing this under $SU(q) \times U(1)$ we find integral $U(1)$ charges for even q and half integral $U(1)$ charges for odd q. Now we can gauge the $U(1)$ factor. This amounts to imposing Gauss law projecting on $U(1)$ invariant states. If q is odd there is no such state. If q is even there are such states. We can also add the Chern-Simons term with coefficient k. Now, the $U(1)$ charge is shifted by k. Therefore, to have states in the Hilbert space we need $k + q/2 \in \mathbf{Z}$.

4. Applications to Type I' Theory

4.1. The Quantum Mechanical System

Let us now consider the larger D0-brane system from which (3.1) is obtained. This consists of n D0-branes propagating on the interval $S^1/\mathbf{Z}_2$. There are also 16 D8-branes in the interval; let us call their positions m_r. In [12] the fields and tree-level Lagrangian for the $Spin(8)$ $N = 8$ theory which occurs in this type I' context were determined. It has gauge group $Spin(n)$. One finds quantum-mechanical coordinates

$$A_{0,9}^{IJ}, \quad X_{1,\ldots,8}^{IJ}, \quad \text{and} \quad x_{1,\ldots,8}$$

where the A^{IJ} are antisymmetric in the $Spin(n)$ indices I and J while the X^{IJ} are in traceless symmetric representations and the xs are singlets. The fermionic superpartners are

$$\lambda_a^{IJ}, \quad \theta_{\dot{a}}^{IJ}, \quad \text{and} \quad s_{\dot{a}}$$

with λ_a in the adjoint, $\theta_{\dot{a}}$ in the traceless symmetric, and $s_{\dot{a}}$ a singlet. Here a is an index in the $\mathbf{8}_s$ of $Spin(8)$ and $\dot{a}$ is an index in the $\mathbf{8}_c$.

This system, like the theory (3.1), has different numbers of bosons and fermions [12]. Let us take n even and calculate the vacuum energy out on the classical flat direction in which $Spin(n)$ is broken to $U(n/2)$. This describes a cloud of zero branes away from the orientifold plane (we are ignoring the D8-branes for now). The $U(1)$ factor in $U(n/2)$ encodes the motion of the zero branes away from the orientifold plane. Let us denote by ϕ the scalar in this multiplet (it is a component of A_9). The vacuum energy generated at one loop (i.e. the zero-point energy) is

$$\Lambda_0 = 8n\phi \tag{4.1}$$

The dynamics of the $U(1)$ multiplet is effectively given by a Lagrangian of the form (3.1).

Let us now consider the effects of the 16 D8 branes. This introduces degrees of freedom coming from the 0-8 strings. Analysis of the worldsheet theory of the 0-8 strings as in §4.2 of [14] reveals that the Neveu-Schwarz sector has vacuum energy $+\frac{1}{2}$, so the lightest states are the Ramond-sector states. Therefore, we must include fermions in the fundamental of $Spin(n)$

$$\chi_r^I, \quad \bar{\chi}_r^I \quad r = 1, \ldots, 16.$$

Their Lagrangian contains terms involving the χs

$$\mathcal{L}_\chi = -i\bar{\chi}_r\dot{\chi}_r - \bar{\chi}_r A_9 \chi_r - \bar{\chi}_r A_0 \chi_r - m_r \bar{\chi}_r \chi_r. \tag{4.2}$$

where the bare masses m_r describe the distance of the D0-branes from the rth D8-brane.

These fields contribute to the zero-point energy as well. If 8 D8-branes sit at one orientifold ($m_r = 0$), then the $\chi s'$ contribution to the vacuum energy is

$$\Lambda_\chi = -8n\phi, \tag{4.3}$$

cancelling (4.1). Thus, eight complex or equivalently sixteen real χ fields are necessary to cancel the linear potential. More generally, the potential is piecewise linear with singularities at $\phi = m_r$. Clearly, the coefficient of the Chern-Simons term is an integer and cannot be renormalized except at one loop. Since it is related by supersymmetry to the potential, the latter also cannot be renormalized.

It is important to stress that this potential does not lead to explicit supersymmetry breaking. The effective Lagrangian with the potential included is supersymmetric and supersymmetry is only spontaneously broken. This is achieved by adding the Chern-Simons term as in (3.1).

This piecewise linear function is the same one which appeared in the space time analysis of [20,13,14] and in the four brane discussion in [6]. In space time it appeared at closed string tree level. On the four brane probe it renormalized the metric at one loop. This is one loop of open strings which is the same as a closed string tree diagram [5]. In our problem it appears as a potential for the zero branes. It is amusing that the same function affects different terms in the two different probes: the metric in the four brane probe and the potential in the zero brane probe. The similarity between them is that these two terms are related by supersymmetry to a Chern-Simons term. Also, these two terms are controlled by a nonrenormalization theorem and therefore this piecewise linear function is exact.

This system is subject to the anomaly constraint (3.6). A similar anomaly has already appeared in three dimensions in [21,22] and in five dimensions in [23,24]. In fact, the analysis in five dimensions is closely related to ours. There the five dimensional theory occurs on a D4-brane probe in the system of D8-branes. Our analysis applies to a zero brane probe of the same underlying spacetime physics.

4.2. The Kinetic Terms

As explained in §2.1, the kinetic terms for ϕ and its fermionic partners λ_a are largely unconstrained by supersymmetry and depend on an arbitrary function $f(\phi)$. At tree level in the type I' system we have $f_{tree} = \frac{1}{\lambda_{I'}}$. The one-loop contribution is proportional to $\frac{1}{\phi^3}$ by dimensional analysis. Direct computation yields a nonzero contribution. This has been previously computed[2] in [12]. This gives an example of a situation where dimensional reduction (in this case from the $(0,8)$ $2d$ theory discussed above) does not preserve a nonrenormalization theorem. In particular, higher-loop contributions are not ruled out (and presumably contribute). The loop expansion, which is an expansion in $\lambda_{I'}/\phi^3$, is sensible for large ϕ. But for $\phi > \frac{1}{\sqrt{\alpha'}}$, we must also consider the effects of the open string oscillator modes which exist in the type I' string theory.

The χs do not run in the loops which contribute to this metric. So even when 8 D8-branes are at either end of the interval, the metric is nontrivial. This is the configuration in which the spacetime dilaton is constant. As we will discuss more fully below, the constant dilaton is reflected in the quantum mechanics by the fact that the contributions (4.1) and (4.3) to the potential cancel. It is surprising that the *metric* for ϕ and X is not the same as the spacetime metric found in [20,13,14]. On the fourbrane probe [6], the metric is constant in this limit. So we we see here a case in which different probes have have qualitatively different Lagrangians in the same background.

4.3. Lorentz Invariance and the Supersymmetry Algebra

The Hamiltonian on the ϕ branch of the $Spin(2)$ theory resulting from the corrections computed above takes the form

$$H = \left(\frac{1}{\lambda_{I'}} + \frac{c}{\phi^3} + \mathcal{O}(\lambda_{I'}) \right)^{-1} P_\phi^2 + \left(16\phi - \sum_{r=1}^{16} |\phi - m_r| \right) + \frac{\lambda_{I'}}{2} P_i^2 + \frac{1}{\lambda_{I'}}. \qquad (4.4)$$

---

[2] We find that the χs do not contribute to f.

Here P_ϕ is the momentum conjugate to ϕ, and P_i is the momentum conjugate to the center of mass position x_i in the transverse 8 dimensions.

The last term is the bare mass of the D0-brane. For large bare mass M, the Hamiltonian for a particle takes the form

$$H_0 = M + \frac{\vec{P}^2}{2M}. \tag{4.5}$$

Our theory (4.4) satisfies this at the classical level with $M = \frac{1}{\lambda_{I'}}$. We must check that the constant piece in the potential, as well as the linear term analyzed above, is consistent with supersymmetry. In this case the key is to keep track of the central term Z in the algebra (3.4). Although Z appears indistinguishable from H in the algebra (3.4), it transforms as a singlet under spacetime $Spin(8,1)$ Lorentz transformations. The term M in H is cancelled by a term $-M$ in Z, leaving $\{Q_a, Q_b\}$ free of the constant piece.

The nontrivial metric generated by higher loop corrections spoils the simple form (4.5). This is not a contradiction with Lorentz invariance since the $S^1/\mathbf{Z}_2$ direction breaks Lorentz invariance explicitly. What is surprising is that the nontrivial metric is generated even if 8 D8-branes lie at each fixed point: the nonvanishing graphs do not involve the fields χ. This is the configuration for which the dilaton tadpole in the spacetime theory cancels locally [20].

For more generic D8-brane positions, the type I' dilaton varies over the interval. On a sublocus of the moduli space, which corresponds in the dual heterotic string to the enhanced gauge symmetry locus in Narain moduli space, the type I' dilaton blows up at the orientifold plane ($\phi = 0$) [20]. This suggests [13] that the enhanced gauge bosons in the type I' theory are bound states of D0-branes localized at $\phi = 0$. The D0-brane mass is $\frac{1}{\lambda_{I'}}$, and this goes to zero at $\phi = 0$ on the enhanced symmetry locus. This is consistent with the linear potential we found within the quantum mechanics, which causes the D0-brane to roll toward the orientifold plane. As the D0-brane approaches $\phi = 0$, it becomes relativistic and the approximation (4.5) breaks down. In this regime, instead of starting from (4.4), one should presumably work with the Dirac-Born-Infeld action to obtain the correct relativistic energy-momentum-mass relation $E = \sqrt{\vec{p}^2 + M^2}$. In higher-dimensional probe theories [6,7], the enhanced symmetry locus in spacetime corresponds to nontrivial renormalization-group fixed points on the probe at the orientifold plane. The analogue of that statement in the $d = 1$ case is the presence of a BPS state with gauge boson quantum numbers corresponding to the global symmetry on the probe. It would be very interesting to make this explicit in the quantum mechanics, though the importance of pair creation in the relativistic regime may render this difficult.

5. Applications to the Matrix Model

One interesting issue in the proposed matrix model formulation of M theory [25] is the origin of spacetime gauge bosons. One approach (as in §9 of [25]) is to extrapolate BPS configurations in a given compactification of the type IIA theory to strong coupling, relying on nonrenormalization theorems and time dilation in the infinite momentum frame to argue that higher order corrections to the weak coupling physics are negligible. This was applied to the $S^1/\mathbf{Z}_2$ case with all the D8-branes at the ends of the interval in [15]. One finds using S and T dualities that the gauge bosons arise from bound states of D0-branes with the D8-branes at the ends of the interval. In particular, bound states of odd numbers of D0-branes fall in the **128** (spinor) of the $Spin(16)$ living on the D8-branes, while bound states of even D0-brane number fall in the **120** of $Spin(16)$. The quantum mechanical analysis of the bound states was extended in [17], where for finite radius the role of winding open strings was explained.

This appears consistent as far as the states go, but our results here suggest difficulties with the more general extrapolation to strong coupling. In the $N = 16$ supersymmetric quantum mechanics describing D0-branes in the type IIA theory, the metric is constrained to be flat by supersymmetry, and there is potentially a nonrenormalization theorem for the $(velocity)^4$ term describing scattering [5,25]. We see from the supersymmetry structure explained in §2 -§4 that the D0-brane system in type IIA on $S^1/\mathbf{Z}_2$ is already unconstrained at the level of the metric or $(velocity)^2$. (Recently, renormalization of $(velocity)^4$ in a system with 8 supercharges was observed in [26].) In the matrix model context, the $N = 8$ quantum mechanics we have studied is supposed to represent a Horava-Witten domain wall in eleven dimensional spacetime. This is only possible when sixteen real χ fields are included in the Lagrangian, since otherwise there is a linear potential due to the wall which spoils locality and translation and rotation invariance far from it. However, we have shown that even in this case there is a nontranslationally invariant correction to the metric in the effective Lagrangian far from the wall. In each order of perturbation theory this metric falls off with distance, but it is easy to see that higher powers of inverse distance are accompanied by higher powers of N. Thus, in the large N limit, it is only for $r \gg N^{1/3}l_{11}$ that we can be confident that the effects of the wall are negligible.

This is precisely analogous to the discussion of the coefficient of the $(velocity)^4$ interaction in [25] . There, a nonrenormalization theorem was conjectured to resolve the problem, but in the present context this cannot be the resolution. Thus, if the $Spin(N)$

matrix quantum mechanics is to represent M theory in the presence of a domain wall, there must be some other principle (presumably valid only at large N) which guarantees that the effects of the wall fall off with distance. This principle might also be valid in the original matrix model, eliminating the necessity of invoking an unproven nonrenormalization theorem. We must also admit the possibility that these arguments show that the matrix models do not give a correct description of eleven dimensional physics.

The fact that different probes of the same spacetime have different moduli space metrics should not disturb us in general. The dilaton and metric may couple differently to different probes. There can even be spacetime curvature terms which appear in the moduli space metric of some probes [27]. However, it is somewhat confusing that in the case where the spacetime equations of motion lead us to expect a flat space with constant dilaton, zero branes and fourbranes seem to exhibit different physics. The nontrivial metric on the moduli space of zero branes does not seem to have an analog in four brane physics. Recall however that the physics of the metric on the four brane world volume field theory maps into a potential in the zero brane quantum mechanics. Perhaps the nontrivial metric of the zero branes maps similarly into some higher dimension term in the four brane Lagrangian. Then we would be able to keep a notion of an underlying spacetime physics which is simply perceived differently by different branes.

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